Math 60 8.6 Compound Inequalities, Day 2

Objectives
6) Solve inequalities using "and"
7) Solve double inequality
8) Solve inequalities using "or"

Math 60 8.6 Compound Inequalities Day 2

(1) Solve $2x-5>-1$ and $4x+2\leq 16$

Graph solution.
Write solution in set notation
Write solution in interval notation

$2x-5>-1$ and $4x+2\leq 16$

Isolate x in each inequality separately

$2x>4$ add 5

$x>2$ div 2

$\underbrace{\quad}_{A}$

AND = $\cap$

$4x\leq 14$ subtract 2

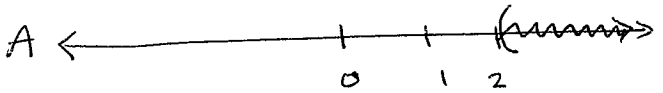
$x\leq \frac{14}{4}$ div 4

$x\leq \frac{7}{2}$ reduce

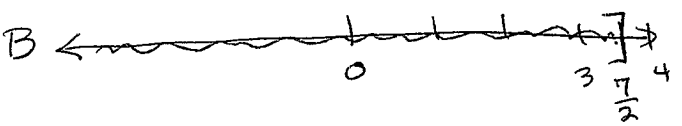
$\underbrace{\quad}_{B}$

Find $A\cap B$.

Graph

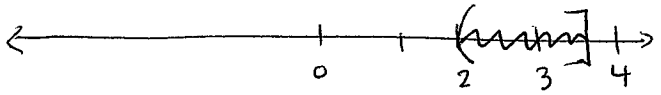
A 

$x>2$

B 

$x\leq \frac{7}{2}$ $\frac{7}{2}=3.5=3\frac{1}{2}$

"And" means intersection: (area shaded in both graphs)
(values in A and in B)

$A\cap B$ 

Graph of solution

Write solution

$\{x \mid 2<x\leq \frac{7}{2}\}$ set notation

$(2,\frac{7}{2}]$ interval notation

Math 60 8.6 Day 2

② Solve $-3x+4>-5$ and $\frac{1}{2}x+7\leq 5$

Graph solution.

Write solution in set notation

Write solution in interval notation

Isolate x in each inequality separately:

$-3x+4>-5$ and $\frac{1}{2}x+7\leq 5$

$-4\quad -4$ $\qquad$ $-7\quad -7$ $\qquad$ subtract 7

$\frac{-3x}{-3} > \frac{-9}{-3}$ $\qquad$ $\frac{1}{2}x\leq -2$ $\qquad$ mult by 2

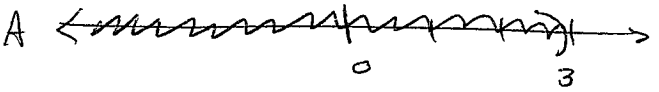
div by neg

$x<3$ $\qquad$ and $\cap$ $\qquad$ $x\leq -4$

A $\qquad$ B

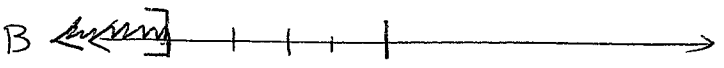
Find $A\cup B$

Graph A



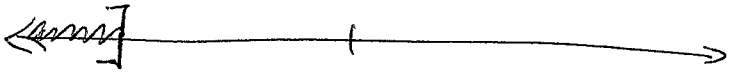
$x<3$

B



"And" means intersection (area shaded in both graphs)
(values in A and in B)

$A\cup B$



graph of solution

Write solution

$\{x \mid x\leq -4\}$ $\qquad$ set notation

$(-\infty, -4]$ $\qquad$ interval notation

Math 60 8.6 Day 2

(3) Solve $\frac{3}{2}x - 2 > 1$ and $-(x+2) \geq 2$

Graph solution
Write solution in set notation
Write solution in interval notation.

Isolate x in each inequality separately

$\frac{3}{2}x - 2 > 1$

$+2 \quad +2$ add 2

$\frac{3}{2}x > 3$ mult by recip

$x > 3 \cdot \frac{2}{3}$

$x > 2$

A

$-(x+2) \geq 2$

option 1: $\div(-1)$ first

$x + 2 \leq -2$

$-2 \quad -2$

$x \leq -4$

option 2: dist neg first

$-x - 2 \geq 2$

$+2 \quad +2$

$-x \geq 4$

$x \leq -4$

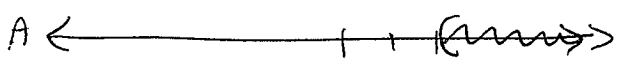
B

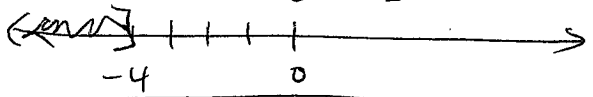
AND $\cap$

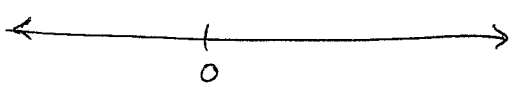
Find $A \cap B$

$x > 2$ and $x \leq -4$

Graph

A  $x > 2$

B  $x \leq -4$

$A \cap B$ 

no overlap

solution $\boxed{\emptyset}$ or $\boxed{\{\}}$ The empty set

Math 60 8.6 Day 2

(4) Solve $0 \le -2x+6 \le 14$

Graph
Write solution in set
Write solution in interval

Method 1: Treat this double inequality as having 3 "sides". When isolating x, do the same to all 3 sides.

$$\underbrace{0}_{\text{1st side}} \le \underbrace{-2x+6}_{\text{2nd side}} \le \underbrace{14}_{\text{3rd side}}$$

A double inequality always means AND

$$0 \le -2x+6 \le 14$$
$$\underline{-6 \qquad -6 \qquad -6}$$

subtract 6

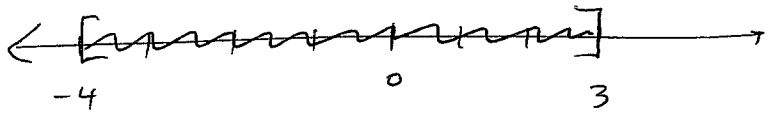
$$\frac{-6}{-2} \le \frac{-2x}{-2} \le \frac{8}{-2}$$

divide by -2
Reverse all inequalities.

$$3 \ge x \ge -4$$

To see more clearly, we can change the order if we swap the direction of the inequalities also.

$$-4 \le x \le 3$$



set notation $\{x \mid -4 \le x \le 3\}$
interval notation $[-4, +3]$

Method 2: Re-write problem as two separate inequalities separated by AND, using middle quantity in both.

$0 \le -2x+6$ and $-2x+6 \le 14$

$$\frac{-6}{-2} \le \frac{-2x}{-2} \qquad \frac{-2x}{-2} \le \frac{8}{-2}$$

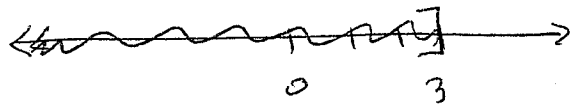
$3 \ge x$ and $x \ge -4$

$\underbrace{x \le 3}_{A}$ and $\underbrace{x \ge -4}_{B}$

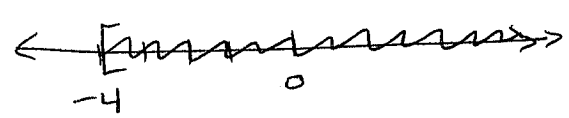
$A \cap B$

cont →

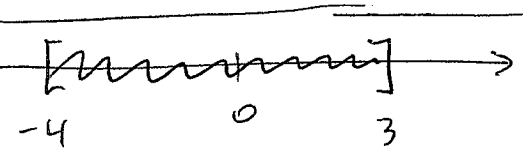
Math 60 8.6 day 2

A 

$x \leq 3$

B 

$x \geq -4$

$A \cap B$ 

overlap = add = intersection

$\{x \mid -4 \leq x \leq 3\}$ set notation

$[-4, 3]$ interval notation

Math 60 8.6 Day 2

(5) Solve $\frac{1}{2}x - 3 < -1$ or $-(6-x) > 1$

Graph solution.
Write solution in set notation.
Write solution in interval notation.

Solve each separately

$\frac{1}{2}x - 3 < -1$ or $-(6-x) > 1$

$+3 \quad +3$

$\frac{1}{2}x < 2$

$x < 2 \cdot 2$

$x < 4$

A

or

option 1: $\div(-1)$ first

$6 - x < -1$

$-6 \quad -6$

$-x < -7$

$-1 \quad -1$

$x > 7$

(4) option 2: dist neg first

$-6 + x > 1$

$+6 \quad +6$

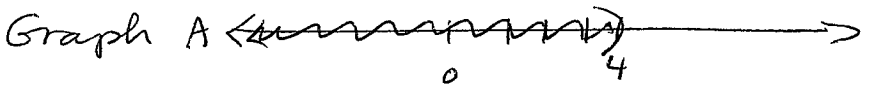
$x > 7$

B

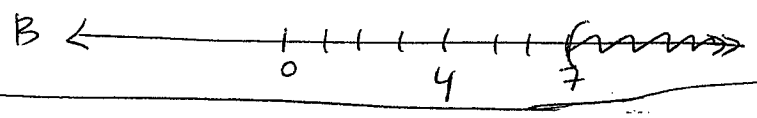
Find A∪B

$x < 4$ or $x > 7$

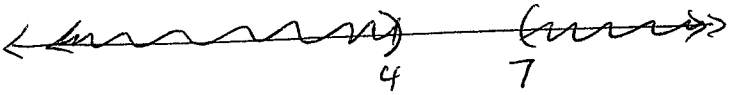
Graph A



B



A∪B



or = union = all shaded values united

set notation $\{x \mid x < 4 \text{ or } x > 7\}$

interval notation $(-\infty, 4) \cup (7, \infty)$

Math 60 8.6 Day 2

⑥ Solve $2-3x \leq 5$ or $-\frac{1}{2}x+1 > -2$

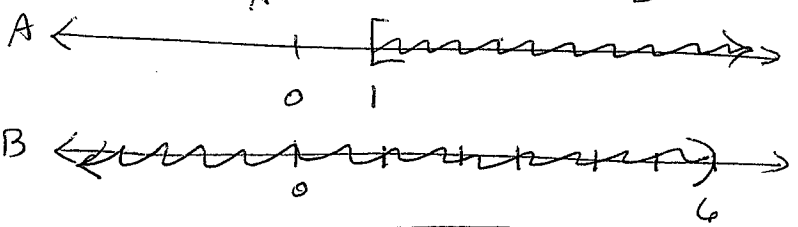
$-2 \quad -2$ $\qquad$ $-1 \quad -1$

$-3x \leq 3$ $\qquad$ $-\frac{1}{2}x > -3$

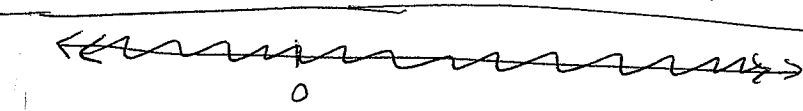
$-3 \quad -3$ $\qquad$ $x < -3 \cdot \frac{-2}{1}$

$x \geq -1$ $\qquad$ $x < 6$

$\underbrace{x \geq 1}_{A}$ or $\underbrace{x < 6}_{B}$



A∪B



all shaded areas united.

set notation $\{x : x \text{ is a real \#}\} = \{x \mid x \in \mathbb{R}\}$

interval notation $(-\infty, \infty)$

⑦ In 2012 a married couple filing jointly who were in the 25% tax bracket paid between \$9735 and \$27735 in federal income taxes.

Federal income tax for this category is \$9735 plus 25% of their taxable income over \$70,700.

Find the range of taxable incomes into which a couple must fall to have been in the 25% tax bracket.

Step 1: What do we need to find? Name a variable.

range of taxable incomes

range means inequality

x = taxable income

Cont →

Math 60 8.6 Day 2

step 2: Translate the math parts of the given information

$9735 \leq$ federal taxes ≤ 27735 { The first sentence }

federal taxes = 9735 + 25% of taxable income over 70,700

{ The second sentence }

step 3: Use variable to translate more:

"taxable income over 70,700"

means

if taxable income is less than 70,700, no percentage is taken.

$\Rightarrow$ subtract 70700 from taxable income

$\Rightarrow (x - 70700)$

step 4: substitute back into translation for federal taxes

federal taxes = 9735 + 25% of $(x - 70700)$

step 5: Translate "25% of" to a basic percent

federal taxes = $9735 + .25(x - 70700)$

step 6: substitute this expression for federal taxes into the inequality

$$9735 \leq 9735 + .25(x - 70700) \leq 27735$$

step 7: Solve the double inequality by doing same to all three "sides" or expressions to isolate x.

$$\underbrace{9735}_{\text{1st "side"}} \leq \underbrace{9735 + .25(x - 70700)}_{\text{2nd "side"}} \leq \underbrace{27735}_{\text{3rd "side}}$$

Math 60 8.6 Day 2

subtract 9735

$$\begin{array}{rcccl} 9735 & \le & 9735 + .25(x-70700) & \le & 27735 \\ -9735 & & -9735 & & -9735 \end{array}$$

$$0 \le 0.25(x-70700) \le 18000$$

option 1: ÷(.25) option 2: distribute(.25)

option 1: divide

$$\frac{0}{0.25} \le \frac{0.25(x-70700)}{0.25} \le \frac{18000}{0.25}$$

$$\begin{array}{rcccl} 0 & \le & x-70700 & \le & 72000 \\ +70700 & & +70700 & & +70700 \end{array}$$

$\$70700 \le x \le \#142700$

option 2: dist

$$\begin{array}{rcccl} 0 & \le & 0.25x - 17675 & \le & 18000 \\ +17675 & & +17675 & & +17675 \end{array}$$

$$\frac{17675}{0.25} \le \frac{0.25x}{0.25} \le \frac{35675}{0.25}$$

$\$70700 \le x \le \#142700$

Math 60: 8.6 Compound Inequalities - Day 2

Objectives, continued:

6) Solve inequalities using "and"
7) Solve double inequalities [same as "and", but different notation]
8) Solve inequalities using "or"

Examples

Solve the compound inequality.

a) Graph the solution.
b) Write the solution set notation.
c) Write the solution in interval notation.

1) $2x-5>-1$ and $4x+2\le 16$

2) $-3x+4>-5$ and $\frac{1}{2}x+7\le 5$

3) $\frac{3}{2}x-2>1$ and $-(x+2)\ge 2$

4) $0\le -2x+6\le 14$

5) $\frac{1}{2}x-3<-1$ or $-(6-x)>1$

6) $2-3x\le 5$ or $-\frac{1}{2}x+1>-2$

7) In 2012, a married couple filing jointly who were in the 25% tax bracket paid between \$9735 and \$27735 in federal income taxes. Federal income tax for this category is \$9735 plus 25% of taxable income over \$70,700. Find the range of taxable incomes into which a couple must fall to have been in the 25% tax bracket.